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Remarks on Relativistic Radiative Reaction

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Abstract

It is shown explicitly that the loss of kinetic energy of a highly relativistic particle in magnetic field is, in general, at the cost of both its transverse and longitudinal motion. Also, in radiation problems it is convenient and appropriate to make a Lorentz transformation into an inertial frame in which the accelerated particle is momentarily at rest.

This note is in response to a recent article in this journal by Sen Gupta (1970). In that article and in an earlier short communication Sen Gupta (1970) had derived an exact integral for a general class of electromagnetic field. This integral, in the case of synchrotron radiation, implies the longitudinal velocity of the particle to be constant. From that he concludes the loss of kinetic energy of a particle in magnetic field is only at the cost of its transverse motion. This statement is in contrast to what we obtained in an earlier paper (Shen, 1970). The discrepancy arises, Sen Gupta asserts, because the instantaneous rest frames associated with the accelerated particle are not inertial frames, so some of the Lorentz transformations used in Shen (1970) to simplify calculations are not justified.

That the longitudinal velocity of a radiating particle is an invariant in a constant magnetic field is entirely correct. We had, although in a less elegant way, also arrived at this result (equation 5-3 of Shen (1970)). This, however, does not imply the longitudinal momentum, and so the longitudinal energy, to be also constant (Shen, 1971). The momentum $\vec{p}$ is given by $\gamma m\vec{v}$, where γ is the energy of the particle in unit of its rest mass. When a particle loses energy through radiation, its longitudinal momentum decreases through the decrease of γ. From $p_\perp = \gamma m v_\perp$, $p_\parallel = \gamma m v_\parallel$ and $v_\parallel =$ constant we have

$$\frac{dp_\perp}{dt} = p_\perp\left(\frac{d\gamma}{dt}\Big/\gamma + \frac{dv_\perp}{dt}\Big/v_\perp\right) \tag{1}$$

$$\frac{dp_\parallel}{dt} = p_\parallel\frac{d\gamma}{dt}\Big/\gamma \tag{2}$$

and

$$\frac{d\gamma}{dt} = \gamma^3 v_\perp \frac{dv_\perp}{dt} \Big/ C^2 \tag{3}$$

Combination of (1), (2), and (3) gives

$$\frac{dp_\parallel}{dt} \Big/ \frac{dp_\perp}{dt} = \frac{p_\parallel}{p_\perp}\left(\frac{\gamma^2 \beta_\perp^2}{1+\gamma^2\beta_\perp^2}\right) \tag{4}$$

hence

$$\frac{dp_\parallel}{dt} \Big/ \frac{dp_\perp}{dt} \approx 0 \qquad \text{for } \beta = \frac{v}{c} \ll 1 \tag{5}$$

and

$$\frac{dp_\parallel}{dt} \Big/ \frac{dp_\perp}{dt} \approx \frac{p_\parallel}{p_\perp} \qquad \text{for } \beta \to 1 \tag{6}$$

Only in the non-relativistic case does the radiation draw all of its energy from the transverse component of the motion, whereas in the ultra-relativistic case the ratio of the longitudinal energy loss to the transverse energy loss is equal to the ratio of the initial values of the two respective components, which also means that the particle's pitch angle maintains its initial value until the particle becomes non-ultra-relativistic. There is no inconsistency between my conclusion that an ultra-relativistic particle injected randomly into a strong magnetic field will lose most of its energy through radiation and that the particle's energy cannot be less than $m_0 C^2(1 - v_\parallel^2/C^2)^{-1/2}$. For example, a particle of initial energy $1000 m_0 C^2$ and initial pitch angle of 30° will reduce its energy to $2m_0 C^2$, not to a substantial portion of its initial energy.

The other point raised by Sen Gupta is that whether it is appropriate to make a Lorentz transformation of the field and other relevant quantities to the 'instantaneous rest frame' of the particle. Since the particle is at acceleration, the 'instantaneous rest frame' is not, contended Sen Gupta, an inertial frame. The transformation may be justified at small acceleration but is no longer valid in the study of strong radiation for which acceleration is necessarily large. This is a rather interesting point. However, the instantaneous rest frame applied in Shen (1970) is not a frame attached to the particle. It is a frame which moves with a (uniform) velocity equal, in magnitude and direction, to the velocity of the particle at the instant of consideration. The particle, although motionless (or nearly motionless if an infinitesimal time apart from the instant), has nevertheless been accelerated in this instantaneous rest frame. (Otherwise, how can one expect the particle to radiate in this frame?) The instantaneous rest frame is by definition an inertial frame irrespective of the magnitude of the accelerations, and the transformation tricks used in Shen (1970) are strictly valid. Such a choice of coordinates, we want to emphasize, is effective only when the consideration is limited to the motion of the particle as a whole. If the particle possesses internal structures (spin, for example) and one wants to study the effect of external field on these internal properties, then the equations of motion which govern the change of these properties must be expressed in the frame attached to the particle. This frame is of course not an inertial frame. As Thomas first pointed out in the derivation of the precession effect bearing his name, the particle's frame rotates with respect to the instantaneous inertial frame with an angular velocity $\vec{\omega}_T = (\gamma - 1)[(\vec{v} \times \vec{a})/v^2]$, where $\vec{v}$ and $\vec{a}$ are the velocity and acceleration of the particle in the observer's frame of reference. For the synchrotron problem the computation become unnecessarily complicated if one wants to carry them out in the particle's frame. Still, it can be easily shown that

$$\left(\frac{d\vec{v}}{dt}\right) \text{particle's frame} = \left(\frac{d\vec{v}}{dt}\right) \text{instantaneous inertial frame } 1 + 0\left(\frac{1}{\gamma}\right) \qquad (7)$$

resulting in no practical difference even in the intermediate steps.

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